The Life and Death of Euler, Bernoulli, Navier-Stokes Equations and Associated CFD for So-called Incompressible Fluid Flow

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Abstract

Euler equations derived in 1755 are the basis for general fluid dynamics analysis. Despite their sparkling beauty, elegance, and sacrosanctity, it is well-known that fluid dynamics is a weird academic area full of paradoxes and disaster solutions, for which, viscosity is often only playing the scapegoat for answering many absurdities [1,2]. In fact, the chastity of Euler and sibling N-S equations has long been contaminated by babysitting them with artificial viscosity, upwinding and other measures in CFD. Although derived by momentum principle, yet no matter how surprisingly and sarcastically, Euler equations are not genuine momentum equations due to incapability to accommodate any cases with energy loss. Then, should we blindly keep worshipping these non-genuine momentum equations forever ? Insightfully, herein we find that the long-lived slippery culprit for this fatal non-genuineness is a seemingly innocent "artificial" equalization of crosswise and streamwise fluid pressures by unawares disregarding any possible energy loss in deriving Euler equations. Although this is a concept-revolution paper with tremendous and panoramic impact on past/future FD/CFD academia, yet its back-to-nature surgery for curing the fatal eulerian error and associated judicious discussion on solution existence/uniqueness are not difficult to understand.

Key Words : Incompressible Flow, FD/CFD, Impact Energy Loss, Historical Concept Revolution, Searching/Curing Fatal Derivation Error

Introduction

Euler equations derived by Leonhard Euler (1707-1783) in 1755 [3] are among the most important equations for general fluid dynamics analysis. Subsequently, during the first half of nineteenth century, viscous effect was involved to obtain the Navier-Stokes (N-S) equations [4,5], named in honor of Claude Louis M. H. Navier (1758-1836) and Sir George Gabriel Stokes (1819-1903). These equations are considered as the frameworks of fluid mechanics. For convenient reference, the traditional Euler equations for inviscid fluid flow in Cartesian coordinate system are written here as

$$-\frac{\partial P}{\partial x}+\rho g_x=\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right) \tag{1}$$

$$-\frac{\partial P}{\partial y}+\rho g_y=\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right) \tag{2}$$

$$-\frac{\partial P}{\partial z}+\rho g_z=\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right) \tag{3}$$