Inherent Numerical Discretization Dilemma Analysis of Euler Equation

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ABSTRACT

Euler equations derived in 1755 are among the most important equations for general fluid analysis. Although this fundamental governing equation looks simple at first glance for incompressible, inviscid fluid flow, it does lead to crucial numerical discretization dilemma related to existence, uniqueness, stability and oscillation problems. Take gradual pipe expansion flow for instance, in the process of deriving Euler equation, the seemingly innocent artificial equalization of crosswise and forward-biased streamwise pressure will give serious discretization error compared with Bernoulli equation, i.e. the wall pressures $P_W = P + dP$ already disappeared from Euler equation will later ghostly bring about troublesome problems, and even violate the fundamental energy principle. Various numerical measures such as artificial viscosity and streamline upwinding are devised awkwardly with the high risk of flow field contamination in an attempt to somehow manipulate some energy dissipation when dealing with incompressible fluid flow. In this research, for the sake of finding the real culprit, we re-scrutinize the original deriving process of Euler equation and insightfully compare the discretized Euler equation with factorized Bernoulli equation, in order to find the inherent true pathology leading to these serious numerical discretization dilemmas of Euler equation.

Key Words: Euler equations, Numerical discretization/CFD, Tracing inherent dilemma, Bernoulli equation factorization.

INTRODUCTION

Fluid dynamics analysis is based on the conservation laws of mass, momentum and energy, interests have been consistently focused on the incompressible fluid flow. Euler equations(1755), as the core equations for FD/CFD were derived by Leonhard Euler only 68 years after Newton's Principia of Mathematica (1687). Afterwards, in the first half of nineteenth century, viscous effect was involved to obtain the Navier-Stokes (N-S) equations [1,2]. These equations are considered as the framework of fluid mechanics. In spite of their glistening elegance and sacrosanct longevity, nevertheless when coming to the numerical solutions of these general equations for incompressible flows, serious difficulties are encountered in the determination of the pressure field, in the fulfillment of the incompressibility condition, in the discretization on nonlinear convection term and so forth. In fact, for many decades, the fundamental theoretical system of fluid dynamics has long been contaminated by adding artificial viscosity or its seemingly more innocent counterpart of upwinding as a daily routine in order to merely get a reluctant solution existent. Meanwhile,