出題負責教師：鄭澤明

建國科技大學 進修部 101 學年度 第 1 學期 班級：二技機械一(必) 科目：工數(1) 期中考試試題

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** 註 班級：機械一甲 學號：10160103 姓名：潘明光

1. 解 $6xy' - y = 0$

$6x\frac{dy}{dx} = y$

$\frac{dy}{y} = \frac{dx}{6x}$

$\int \frac{dy}{y} = \int \frac{dx}{6x}$

$\ln y = \frac{1}{6}\ln x + C$

$e^{\ln y} = e^{\frac{1}{6}\ln x} * e^C$

$y = x^{\frac{1}{6}} \cdot C$

$y^6 = x \cdot C$

2. 解 $y' - \frac{y}{x} + \frac{y^2}{x^2} = 0$ 設 $\frac{y}{x}$ 為 u $y' = u + u'x$

$u + u'x - u + u^2 = 0$

$u'x = -u^2$

$x\frac{du}{dx} = -u^2$

$\frac{du}{u^2} = \frac{-dx}{x}$

$u^{-2}du = -\frac{dx}{x}$

$\int u^{-2}du = \int -\frac{dx}{x}$ $\frac{1}{-2+1}u^{-2+1} = -\ln x + C$

$u^{-1} = \ln x + C$

$\frac{1}{u} = \frac{1}{\frac{y}{x}} = -\ln x + C$

$\frac{x}{y} = \ln x + C$

3. 解 $(4x^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$

設 $4x^3y^3 - 2xy$ 為 M $3x^4y^2 - x^2$ 為 N

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ 為正合

$\frac{\partial M}{\partial y} = \frac{\partial(4x^3y^3 - 2xy)}{\partial y} = 12x^3y^2 - 2x$

$\frac{\partial N}{\partial x} = \frac{\partial(3x^4y^2 - x^2)}{\partial x} = 12x^3y^2 - 2x$

為正合

1. $u = \int M dx = \int (4x^3y^3 - 2xy)dx = x^4y^3 - x^2y + P(y)$

2. $u = \int N dy = \int (3x^4y^2 - x^2)dy = x^4y^3 - x^2y + Q(x)$

總合 1、2 之解

$x^4y^3 - x^2y + C = 0$

$x^4y^3 - x^2y = C$

4. 解 $(2y - 3x)dx + xdy = 0$

設 $(2y-3x)$ 為 M x 為 N $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ 為正合

$\frac{\partial M}{\partial y} = \frac{\partial(2y-3x)}{\partial y} = 2$

$\frac{\partial N}{\partial x} = \frac{\partial x}{\partial x} = 1$

$2 \neq 1$ 非正合

求積分因子 f

$M \times f = M'$ $N \times f = N'$

$\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}$

$\frac{\partial M'}{\partial y} = \frac{\partial f(2y-3x)}{\partial y} = \frac{\partial 2yf - 3xf}{\partial y} = 2f - 3xf_y + 2yf_y$

$\frac{\partial N'}{\partial x} = \frac{\partial fx}{\partial x} = f + xf_x$

$2f - 3xf_y + 2yf_y = f + xf_x$

$f_y(-3x + 2y) + 2f = f + xf_x$ 設 f 為 x 的函數則 $f_y(-3x+2y) = 0$

$2f = f + xf_x$

$f = xf_x = x\frac{df}{dx}$

$\frac{df}{f} = \frac{dx}{x}$

$\int \frac{df}{f} = \int \frac{dx}{x}$

$\ln f = \ln x$

$f = x$

將積分因子帶入

1. $u = \int M' dx = \int (2xy - 3x^2)dx = x^2y - x^3 + P(y)$

2. $u = \int N' dy = \int x^2 dy = x^2y + Q(x)$

總合 1、2 之解

$x^2y - x^3 + C = 0$

$x^2y - x^3 = C$

5. 解 $(x - y^2)dx + 2xydy = 0$

於背面

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建國科技大學 進修部 101 學年度 第 1 學期 班級：二技機械一(必) 科目：工數(1) 期中考試試題

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** 註 班級：進二技机-甲 學號：10160160 姓名：吳祥豪

1. 解 $6xy'-y=0$

$6xy'-y=0$

$\Rightarrow 6\cdot x\cdot y'=y$

$\Rightarrow 6x\frac{dy}{dx}=y$

$\Rightarrow 6\frac{dy}{y}=\frac{dx}{x}$

$\Rightarrow 6\int\frac{1}{y}dy=\int\frac{1}{x}dy$

$\Rightarrow 6\ln|y|+C=\ln|x|+C$

$\Rightarrow y^6=C\cdot x$ #

2. 解 $y'-\frac{y}{x}+\frac{y^2}{x^2}=0$

設 $\frac{y}{x}=u$

$y=xu$

$y'=u+xu'$

$y'-\frac{y}{x}+\frac{y^2}{x^2}=0$

$\Rightarrow u+xu'-u+u^2=0$

$\Rightarrow xu'+u^2=0$

$\Rightarrow x\frac{du}{dx}=-u^2$

$\Rightarrow \frac{dx}{dx}=-\frac{u^2}{du}$

$\Rightarrow \frac{dx}{x}=-\frac{du}{u^2}$

$\Rightarrow \int\frac{dx}{x}dx=-\int u^{-2}du$

$\Rightarrow \ln|x|+C=-(-u^{-1})$

$\Rightarrow \ln|x|+C=u^{-1}$ 代入 $u=\frac{y}{x}$

$\Rightarrow \ln|x|+C=(\frac{y}{x})^{-1}$

$\Rightarrow \ln|x|+C=\frac{x}{y}$ #

3. 解 $(4x^3y^3-2xy)dx+(3x^4y^2-x^2)dy=0$

M N

$\frac{dM}{dy}=\frac{4x^3y^3-2xy}{dy}=12x^3y^2-2x$

$\frac{dN}{dx}=\frac{3x^4y^2-x^2}{dx}=12x^3y^2-2x$

$dM=dN$ (正合)

$u=\int(4x^3y^3-2x)dx$

$\Rightarrow x^4y^3-x^2y+k(y)$

$u=\int(3x^4y^2-x^2)dy$

$\Rightarrow x^4y^3-x^2y+k(x)$

$u=x^4y^3-x^2y=C$ #

4. 解 $(2y-3x)dx+xdy=0$

M N

$\frac{dM}{dy}=\frac{d(2y-3x)}{dy}=2$

$\frac{dN}{dx}=\frac{dx}{dx}=1$ (不合)

(求積分因子)

$\frac{dM'}{dy}=\frac{d[f(2y-3x)]}{dy}=2y\frac{df}{dy}+2f-3x\frac{df}{dx}-f\frac{df}{dx}$

$\frac{dN'}{dx}=\frac{d[f\cdot x]}{dx}=x\frac{df}{dx}+1f$

$\frac{dM'}{dy}=\frac{dN'}{dx}\Rightarrow 2y\frac{df}{dy}+2f-3x\frac{df}{dx}=x\frac{df}{dx}+1f$

$\Rightarrow (2y-3x)\frac{df}{dy}+2f=x\frac{df}{dy}+1f$ 視變項設為0

$\Rightarrow 2f=x\frac{df}{dy}+f\Rightarrow f=x\frac{df}{dx}\Rightarrow\int\frac{f}{df}=\int\frac{x}{dx}\Rightarrow\ln f=\ln x$

$\Rightarrow f=x$ (代入原式)

$(2xy-3x^2)dx+x^2dy=0$

$u=\int(2xy-3x^2)dx$

$\Rightarrow x^2y-x^3+k(y)$

$u=\int x^2dx$

$\Rightarrow x^2y+k(x)$

$u=x^2y-x^3=C$

5. 解 $(x-y^2)dx+2xydy=0$

M N

$\frac{dM}{dy}=\frac{d(x-y^2)}{dy}=-2y$

$\frac{dN}{dx}=\frac{d(2xy)}{dx}=2y$ (不合)

(求積分因子)

$\frac{dM'}{dy}=\frac{d[f(x-y^2)]}{dy}=x\frac{df}{dy}+f\frac{dx}{dy}-y^2\frac{df}{dy}-2yf$

$\frac{dN'}{dx}=\frac{d[f(2xy)]}{dx}=2xy\frac{df}{dx}+2yf+2f\frac{dy}{dx}$

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** 註　班級：＿＿＿＿＿＿　學號：10160014　姓名：黃志銘

1. 解 $6xy'-y=0$

$\Rightarrow 6xy'-y=y$

$\Rightarrow 6x\frac{dy}{dx}-y=0$

$\Rightarrow 6x\frac{dy}{dx}=y$

(積數常數係數) $\Rightarrow \frac{dx}{dx}=\frac{y}{dx}$

$\Rightarrow \frac{dx}{6x}=\frac{dy}{y}$

$\Rightarrow \int\frac{dx}{6x}=\int\frac{dy}{y}$

$\Rightarrow \frac{1}{6}\ln x=\ln y+C$

$\Rightarrow \ln x^{\frac{1}{6}}=\ln y+\ln C$

$=\ln yC$

$6xy'-y=0 \Rightarrow 6x\frac{dx}{dx}-y=0$

$=6x\frac{dy}{dx}=y \Rightarrow \frac{6x}{dx}=\frac{y}{dy}$

$\frac{x}{y^6}=C \Rightarrow x^{\frac{1}{6}}=y \Rightarrow \frac{x^{\frac{1}{6}}}{y}=C$

$y^6=Cx$

2. 解 $y'-\frac{y}{x}+\frac{y^2}{x^2}=0$

Let $\frac{y}{x}=u$ $\therefore y'=\frac{\partial(xu)}{\partial x}=u+xu'$

$\therefore u+xu'-u+u^2=0$

$\Rightarrow xu'+u^2=0$

$\Rightarrow x\frac{du}{dx}=-u^2$

$\Rightarrow u^{-2}du=-\frac{dx}{x}$

$\Rightarrow \int\frac{dx}{x}dx=-\int\frac{du}{u^2}du$

$\Rightarrow \ln|x|+C=-(-u^{-1})$

$\Rightarrow \ln|x|+C=u^{-1}$

$\Rightarrow \ln|x|+C=(\frac{y}{x})^{-1}$

$\Rightarrow \ln|x|+C=\frac{x}{y}$

3. 解 $(4x^3y^3-2xy)dx+(3x^4y^2-x^2)dy=0$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ M N

$\frac{\partial M}{\partial y}=\frac{\partial(4x^3y^3-2xy)}{\partial y}=(3y^2\cdot 4x^3)-2x=12x^3y^2-2x$

$\frac{\partial N}{\partial x}=\frac{\partial(3x^4y^2-x^2)}{\partial x}=12x^3y^2-2x$

① $U=\int M\,dx=\int(4x^3y^3-2xy)\,dx=\frac{4}{3+1}x^{3+1}y^3-\frac{2}{1+1}x^{1+1}y=x^4y^3-x^2y+P(y)$

② $U=\int N\,dy=\int(3x^4y^2-x^2)\,dy=\frac{3}{2+1}x^4y^{2+1}-x^2y=x^4y^3-x^2y+Q(x)$

綜合①、②之解 $U=x^4y^3-x^2y=C$

4. 解 $(2y-3x)dx+xdy=0$ M N $\therefore \frac{\partial M}{\partial x}\neq\frac{\partial N}{\partial y}$ ∴非正合恰積和因子 f

$\frac{\partial M}{\partial y}=\frac{\partial(2y-3x)}{\partial y}=2$ $\frac{\partial N}{\partial x}=1$ 使 $M'=Mf$；$N'=Nf$

$\frac{\partial M'}{\partial y}=\frac{\partial N'}{\partial x}$ 則：$\frac{\partial(2xf-3xf)}{\partial y}=\frac{\partial(xf)}{\partial x}=2f+2yf_y-3xf_y=f+xf_x$

$\Rightarrow f+f_y(2y-3x)=xf_x=x\frac{\partial f}{\partial x}$ 表 $f_y=0$ 既 f 僅為 x 之函數 則：$f=x\frac{\partial f}{\partial x}$

$\therefore$ 積分因子 $f=x$ $\frac{df}{f}=\frac{dx}{x}\Rightarrow \ln f=\ln x\Rightarrow f=x$

則①$U=\int M'dx=\int(2yx-3x^2)dx=yx^2-x^3+P(y)$

②$U=\int N'dy=\int x^2dy=x^2y+Q(x)$ 綜合①、②得 $U=x^2y-x^3=C$

5. 解 $(x-y^2)dx+2xydy=0$

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** 註 班級：机械一甲 學號：~~101601000~~ 101601006 姓名：許雅茹

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1. 解 $6xy'-y=0$

$6xy'=y$

$6x\frac{dy}{dx}=y$

$6\cdot\frac{1}{y}dy=\frac{1}{x}dx$

$\int\frac{6}{y}dy=\int\frac{1}{x}dx$

$6\ln(y)=\ln(x)+C$

$\ln(y)^6=\ln(x)+C$

$e^{\ln(y)^6}=e^{\ln(x)+C}=e^C\cdot e^{\ln(x)}$

$y^6=C\cdot x$ ✓

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2. 解 $y'-\frac{y}{x}+\frac{y^2}{x^2}=0$

Let $\frac{y}{x}=u$

$y'=\frac{\partial(xu)}{\partial x}=u+xu'$

$\therefore u+xu'-u^2=0$

$\Rightarrow xu'+u^2=0$

$\Rightarrow x\frac{du}{dx}=-u^2$ →

→ $u^{-2}du=\frac{-dx}{x}$

$\int u^{-2}du=\int\frac{-dx}{x}$

$\frac{1}{-2+1}u^{-2+1}=-u^{-1}=-\ln|x|+C$

$\therefore\frac{1}{u}=\ln|x|+C$

$\therefore\frac{x}{y}=\ln|x|+C$

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3. 解 $\underbrace{(4x^3y^3-2xy)}_{M}dx+\underbrace{(3x^4y^2-x^2)}_{N}dy=0$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y}=\frac{\partial(4x^3y-2xy)}{\partial y}=(3y^2-4x^3)-2x$

$=12x^3y^2-2x$

$\frac{\partial N}{\partial x}=\frac{\partial(3x^4y^2-x^2)}{\partial x}=12x^3y^2-2x$

→ ① $u=\int M\,dx$

$=\int(4x^3y^3-2xy)dx$

$=\frac{4}{3+1}x^{3+1}y^3-\frac{2}{1+1}x^{1+1}y$

→ ② $u=\int N\,dy$

$=\int(3x^4y^2-x^2)dy$

$=\frac{3}{2+1}x^4y^{2+1}-xy^2$

$=x^4y^3-x^2y+\phi(x)$

綜合①、②之解

$u=x^4y^3-x^2y+C$

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4. 解 $\underbrace{(2y-3x)}_{M}dx+\underbrace{x}_{N}dy=0$

$\frac{\partial M}{\partial y}=\frac{\partial(2y-3x)}{\partial y}=2$

$\frac{\partial N}{\partial x}=\frac{\partial(x)}{\partial x}=1$

$\frac{\partial M}{\partial x}\neq\frac{\partial N}{\partial y}$

非正合，須積分因子 f

使 $M'=Mf$ $N'=Nf$

→ $\frac{\partial M'}{\partial y}=\frac{\partial N'}{\partial x}$

則 $\frac{\partial(2yf-3xf)}{\partial y}=\frac{\partial(xf)}{\partial x}$

$\Rightarrow 2f+2yf_y-3xf_y=f+xf_x$

$f+f_y(2y-3x)=xf_x=x\frac{\partial f}{\partial x}$

若 $f_y=0$ 即 f 僅為 x 函數

→ 則 $f=x\frac{\partial f}{\partial x}$

$\frac{\partial f}{f}=\frac{\partial x}{x}\Rightarrow\ln f=\ln x\Rightarrow f=x$

積分因子 $f=x$

則 ① $u=\int M'dx$

$=\int(2yx-3x^2)dx$

$=yx^2-x^3+P(y)$

② $u=\int N'dy$

$=\int x^2dy=x^2y+Q(x)$

∴綜合①、②得 $u=x^2y-x^3=C$

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5. 解 $\underbrace{(x-\frac{1}{x^2+y^2})}_{}dx+2xydy=0$

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** 註 班級：機械四甲 學號：98801408 姓名：許銘峰

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1. 解 $6xy' - y = 0$

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2. 解 $y' - \frac{y}{x} + \frac{y^2}{x^2} = 0$

Let $\frac{y}{x} = u$

$y' = \frac{\partial(xu)}{\partial x} = u + xu'$

$\therefore u + xu' - u + u^2 = 0$

$\Rightarrow x\frac{du}{dx} = -u^2$

$\Rightarrow u^{-2}du = -\frac{dx}{x}$

$\Rightarrow \int u^{-2}du = \int -\frac{dx}{x}$

$\Rightarrow \frac{1}{-1+1}u^{-1+1} = -u^{-1} - \ln x + C$

$\therefore \frac{1}{u} = \ln x + C$

$\therefore \frac{x}{y} = \ln|x| + C$

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3. 解 $(4x^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = \frac{\partial(4x^3y^3 - 2y)}{2y} = (3y^2 - 4x^3) - 2x$

$\frac{\partial N}{\partial x} = \frac{2(3x^4y^2 - x^2)}{2x} = 12x^3y^2 - 2y$

① $u = \int M dx$

$= \int (4x^3y - 2xy)dx$

$= \frac{4}{3+1}x^{1+1}\frac{2}{1+1}x^{1+1}y$

$= \frac{3}{2+1}x^4y^{2+1} - x^2y$

$= x^4y^2 - x^2y + Q(x)$

③ $u = \int N dy$

$= \int (3x^4y^2 - x^2)dy$

① ② 之解

$u = x^4y^3 - x^2y = C$

4. 解 $(2y - 3x)dx + xdy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial(2y-3x)}{\partial y} = 2$

$\frac{\partial N}{\partial x} = \frac{\partial(x)}{\partial x} = 1$

$\therefore \frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

$\therefore$ 非恰須積因子 f

使 $M' = Mf$; $N = Nf$

$\frac{\partial M'}{\partial y} = \frac{\partial N}{\partial x}$

則 $\frac{\partial(2yf - 3xf)}{\partial y} = \frac{\partial(xf)}{\partial x}$

$\Rightarrow 2f + 2yf_y - 3xf_y = f + xf_x$

$\Rightarrow -f + f_y(2y - 3x) = xf_x = x\frac{df}{dx}$

若 $f_y = 0$ 即 f 僅為 x 之函數

則 $f = x^{\frac{2f}{\partial x}}$

$\frac{df}{f} = \frac{\partial x}{x} \Rightarrow \ln f = \ln x \Rightarrow f = x$

$\therefore$ 積分因子 $f = x$

則 $u = fM'dx$

$= \int (2yx - 3x^2)dx$

$= yx^2 - x^3 + P(y)$

② $u = \int N' dy$

$= \int x^2 dy$

$= x^2y + Q(x)$

①② 得

$u = x^2y - x^3 = C$ //

5. 解 $(x - y^2)dx + 2xydy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial(x+y^2+x)}{\partial y} = 2y$ X

$\Rightarrow \frac{\partial N}{\partial y} = \frac{\partial(xy)}{\partial x} = y$ X

$M \cdot f = M' \Rightarrow \frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}$

$N \cdot f = N'$

$\frac{\partial M'}{\partial y} = \frac{fx^2 + fy^2 + f(x)}{2y}$

$\Rightarrow x^2f_y + 2f + 2yf_y + f_y + xf_x$

$\frac{\partial M'}{\partial x} = \frac{\partial(fy+fy)}{\partial x} = xyf_x + f_y$

$fy(x^2+y^2+x) + fy = xy\frac{df}{dx}$

$f = x^{\frac{2f}{\partial x}}$

$f = x\frac{\partial f}{\partial x} \Rightarrow \int \frac{dx}{x} = \int \frac{df}{f}$

$\Rightarrow \ln x = \ln f$

① $u = f_{M'}dx$

$= \int (x^3 + xy^2 + x^2)dx$

$= \frac{1}{4}x^4 + \frac{1}{2}x^2y + \frac{1}{3}x^3 + P(y)$

② $u = \int N' dy$

$= \int (x^2y)dy$

$= \frac{1}{2}x^2y^2 + Q(x)$

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** 註 班級：土木三甲 學號：9940407 姓名：許佑豪 (重修生)

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1. 解 $6xy'-y=0$

Ans: $6xy'-y=0$

$6xy'=y$

$6x\cdot\frac{dx}{dx}=y$

$6\cdot\frac{1}{y}dy=\frac{1}{x}dx$

$\int\frac{6}{y}dy=\int\frac{1}{x}dx$ $\int\frac{6}{y}=\int\frac{1}{x}dx$

$6\ln(y)=\ln(x)+C$

$\ln(y^6)=\ln(x)+C$

$e^{\ln(y^6)}=e^{\ln(x)+C}$

$=e^C\cdot e^{\ln(x)}$

$y^6=e\cdot x$

2. 解 $y'-\frac{y}{x}+\frac{y^2}{x^2}=0$

Ans: $\frac{x}{y}=\ln x+C$

令 $\frac{y}{x}=u$; $y=xu$

$y'=u+xu'$

$u+xu'-u+u^2=0$

$xu'=-u^2$

$x\cdot\frac{du}{dx}=-u^2$

$-\frac{1}{u^2}du=\frac{1}{x}dx$

$-\int u^{-2}du=\int\frac{1}{x}dx$

$\frac{1}{u}=\ln(x)+C$ $\therefore\frac{x}{y}=\ln x+C$

3. 解 $(4x^3y^3-2xy)dx+(3x^4y^2-x^2)dy=0$

Ans: $x^4y^3-xy^2=C$

$u=\int n\,dy=\int(3x+y-1)dy$

$=3xy+\frac{1}{1+1}y-y+\phi(x)$

$=3xy+\frac{1}{2}x^2-y+\phi(x)$

$\therefore u=3xy+\frac{1}{2}x^4+\frac{1}{2}y^2-y+C$

$\therefore 3xy+\frac{1}{2}x^4+\frac{1}{2}y^2-y=C$

4. 解 $(2y-3x)dx+xdy=0$

$M=2y-3x$, $\frac{\partial M}{\partial y}=2$ ✓

$N=x$, $\frac{\partial N}{\partial y}=1$ ✓

找出積分因子 $=Dx$

?

$M=2xy-3x^2$, $\frac{\partial M}{\partial y}=2x$

$N=x^2$, $\frac{\partial N}{\partial x}=2x$

$\int M\,dx=2\cdot\frac{x^2}{2}\cdot y-3\cdot\frac{x^3}{3}=x^2y-x^3+(C_1)$ 正合

$\int N\,dy=x^2y+(C_2)$

$x^2y-x^3=C$

5. 解 $(x-y^2)dx+2xydy=0$

$x-y^2$

$\frac{M}{dy}=\frac{d(x-y^2)}{dy}=-2y$

$\frac{N}{dx}=d\frac{(2xy)}{dx}=2y$ 相等 存在 正合

$u=\int(x-y)^2dx$

$=\frac{1}{1+1}x^{1+1}-xy^2+C$

$\frac{1}{2}u=x^2y^2+C$

$u=\int 2xy\,dy$

$=2x\frac{1}{1+1}+C$

$=xy^2+C$

正解 $xy^2+\frac{1}{2}x^2=C$