106 出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二乙 學號：99401099 姓名：張志銘

A. Terminology (15%)

independent variable, general solution, initial condition

自變數，不隨別的變數改變之變數

通解，滿足DE之代數函數關係式

初始條件，由通解求特解，自變數被指定在某一點之約束條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 3, 1 線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 2, 1, 線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 2, 3 非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$e^{2x} \cdot 2$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2 + 3$

$dy = (2x^2+3)\,dx$

$\int$

$y = \frac{2}{3}x^3 + 3x + C$

2. $(\cos x)dy = y \sin x\, dx$

$\frac{1}{y}dy = \frac{\sin x}{\cos x}dx$

$\int$

$\Rightarrow \ln y = -\int \frac{1}{u}du$

令 $u = \cos x$

$du = -\sin x\, dx$

$\ln y + \ln \cos x = C$

$\ln y = -\ln u$

$\ln y = -\ln \cos x$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$4x^3y^3dx = -(3x^4y^2dy)$

$\frac{4x^3}{3x^4}dx = -(\frac{y^2}{y^3}dy)$

$\int$

$= \frac{4}{3}\ln x = -\ln y + C$

$\frac{4}{3}\ln x + \ln y = C$

$\frac{4}{3}(\ln xy) = C$

$\ln x^4 + \ln y^3 = C$

$x^4y^3 = C$

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

M N

$\frac{\partial M}{\partial y} = 3$, $\frac{\partial N}{\partial x} = 3$

$U_1 = \int (x+3y)dx$

$= \frac{1}{2}x^2 + 3xy + C$

$U_2 = \int (3x - 5y + 2)dy$

$= 3xy - \frac{5}{2}y^2 + 2y + C$

$U = \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y + C$

5. $xyy' - y^2 + x^2 = 0$

good!

令 $y = ux$, $y' = u'x + u$

$x(ux)(u'x+u) - u^2x^2 + x^2 = 0$

$xuu' + u^2 - u^2 + 1 = 0$

$xu\frac{du}{dx} = -1$

$udu = -\frac{dx}{x}$ $u = \frac{y}{x}$

$\frac{1}{2}u^2 = -\ln x + C$

$\frac{1}{2}\frac{y^2}{x^2} + \ln x = C$

$\frac{y^2}{x^2} + 2\ln x = C$

$\frac{y^2}{x^2} + \ln x^2 = C$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

M N

$\frac{\partial N}{\partial y} = 2y$, $\frac{\partial N}{\partial x} = y$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y - y}{N} = \frac{y}{xy} = \frac{1}{x}$

$I = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

乘入原式

$\int (x^4 + xy^2 + x)dx$

$= \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2$

$\int x^2y\,dy$

$= \frac{1}{2}x^2y^2 + C$

$\frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C$

7. $y' + \frac{1}{x}y = x^2$

$y' + Py = Q$

$I = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$y = \frac{1}{I}[\int IQ\,dx + C]$

$\frac{1}{x}[\int f(x)\cdot x^3\,dx + C]$

$\frac{1}{x}[\int x^4\,dx + C]$

$\frac{1}{x}[\frac{1}{5}x^5 + C]$

$y = \frac{1}{5}x^4 + \frac{C}{x}$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機二乙 學號：9940110I 姓名：蘇鈺涵

A. Terminology (15%)

independent variable, general solution, initial condition

1. 自變數：不隨別人改變而改變的變數。
2. 通解：滿足 DE之代數函數關係式，亦即原函數該DE之，一n階 ODE的解包含n個任意常數
3. 初始條件：由通解求特解，自變數被指定在某一點之拘束條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 三階，一次，X線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二，一，非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二，三，非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$2e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$dy = (2x^2+3)dx$

$y = \frac{2}{3}x^3 + 3x + C$

( ) y -1

( )

2. $(\cos x)dy = y\sin x\,dx$

$\int \frac{dy}{y} = \int \frac{\sin x}{\cos x}dx$

令 $u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int \frac{-du}{u} = -\ln|\cos x|$

$\ln|y| = -\ln|\cos x| + \ln|C|$

$y = \frac{C}{\cos x}$ ✓

3. $4x^3y^3dx + 3x^4y^2dy = 0$

M N

$\frac{\partial M}{\partial y} = 12x^3y^2$

$\frac{\partial N}{\partial x} = 12x^3y^2$ } 正合

$u_1 = \int 4x^3y^3dx = x^4y^3 + C$

$u_2 = \int 3x^4y^2dy = x^4y^3 + C$

$u = x^4y^3 + C$

4. $(x+3y)dx + (3x-5y+2)dy = 0$

$\frac{\partial}{\partial y}(x+3y) = 3$

$\frac{\partial}{\partial x}(3x-5y+2) = 3$ } 正合

$u = \frac{1}{2}x^2 + 3yx - \frac{5}{2}y^2 + 2y$

$u_1 = \int (x+3y)dx$

$= \frac{1}{2}x^2 + 3yx + C$

$u_2 = \int (3x-5y+2)dy$

$= 3xy - \frac{5}{2}y^2 + 2y + C$

5. $xyy' - y^2 + x^2 = 0$

$y = ux$, $y' = u + xu'$

$x(ux)(u+xu') - u^2x^2 + x^2 = 0$

$u^2 + xuu' - u^2 + 1 = 0$

$\int xu\frac{du}{dx} + 1 = 0$

$\int u\,du = \int -\frac{dx}{x}$

$\frac{1}{2}u^2 = -\ln x + C$

$u^2 = -2\ln x + \ln$

$u = \frac{y}{x}$

$\frac{y^2}{x^2} = -\ln x^2 + \ln$

M N

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^3 + y^2 + 1) = 2y$

$\frac{\partial M}{\partial x} = \frac{\partial}{\partial x}(xy) = y$ } 非正合

$\frac{2y - y}{N} = \frac{y}{xy} = \frac{1}{x} = f(x)$

$e^{\int f(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$x^4 + xy^2 + xdx + x^2ydy = 0$

$u_1 = \int (x^4 + xy^2 + x$

$= \frac{1}{5}x^5 + \frac{1}{2}$

$u_2 = \int x^2y\,dy = \frac{1}{2}$

$u_1 \cup u_2$

$= \frac{1}{5}x^5 + \frac{1}{2}x^2$

$+ \frac{1}{2}$

$+ C$

7. $y' + \frac{1}{x}y = x^2$

P Q

$\frac{dy}{dx} + \frac{1}{x}y = x^2$

$e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$y = \frac{1}{I}[\int I \cdot Q\,dx + C]$

$= \frac{1}{x}[\int x \cdot x^2dx + C]$

$= \frac{1}{x}[\int x^3dx + C]$

$= \frac{1}{x}[\frac{x^4}{4} + C]$

$= \frac{x^4}{4x} + \frac{C}{x}$

$= \frac{x^3}{4} + \frac{C}{x}$

出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期中考) 考試試題

** 註：關書、不可使用計算機 班級：機二乙 學號：9940 1106 姓名：戴克駿

A. Terminology (15%)

independent variable, general solution, initial condition

(自變數) (通解) (初始條件)

不 改 變 的 一 、

凡 是 滿 足 此 式 之 函 數 關 係 式

由 GS 求 PS，自 變 數 代 入 某 一 時 刻 之 值

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 3階一次 非线性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階一次 非线性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階三次 非线性

C. Complete the following table(10%)

f	f'	f	f'
sinx	cosx	e^{2x}	$\frac{1}{2}e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2 + 3$

$dy = (2x^2+3)\,dx$

$\int dy = \int (2x^2+3)\,dx + c$

$y = \frac{2}{3}x^3 + 3x + x + c$

2. $(\cos x)dy = y \sin x\, dx$

$\frac{dy}{y} = \frac{\sin x\, dx}{\cos x}$ 令 $u = \cos x$

$du = -\sin x\, dx$

$\ln|y| = \int \frac{\sin x\, dx}{\cos x}$

$\ln|y| = \int \frac{-du}{u} \Rightarrow \ln|y| = -\ln|u|$

$\Rightarrow \ln\frac{u}{y} = -\ln\frac{\cos x}{y}$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

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4. $(x+3y)dx + (3x-5y+2)dy = 0$

M N

$\frac{\partial M}{\partial y} = 3$ $\frac{\partial N}{\partial x} = 3 \Rightarrow$?

5. $xyy' - y^2 + x^2 = 0$

$xy(\frac{dy}{dx}) - y^2 + x^2 = 0$

$xy(\frac{dy}{dx}) = -(y^2+x^2)$

$xy\,dy = -(y^2+x^2)\,dx$

$y = ux$ $y' = u'x+u$

$x(ux)(u'x+u) = ux^2 + x^2\,dx$

$xu(u'x)+u^2 = u+1\,(dx)$

$y(\frac{dy}{dx}) + \frac{y^2}{x^2} = \frac{y}{x} + 1$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

M N

$\frac{\partial M}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = y$ ✓

$I = e^{\int p(x)\,dx} = e^{\int \frac{1}{x}dx} = x$ ✓

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{2y-y}{N} = \frac{1}{x}$ (ln x) $\Rightarrow$?

7. $y' + \frac{1}{x}y = x^2$

P Q

$I = e^{\int p(x)\,dx}$

$= e^{\int \frac{1}{x}dx}$

$= x$

$y = \frac{1}{I}[\int IQ\,dx + c]$

$= \frac{1}{I}[\int (x)(x^2)\,dx + c]$

$= \frac{1}{x}[\frac{1}{4}x^4 + c]$

$= \frac{1}{4}x^3 + \frac{c}{x}$

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出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二乙 學號：9940111Y 姓名：蔡相芭.

A. Terminology (15%)

independent variable, general solution, initial condition

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B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 三階一次線性方程式.

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階一次非線性方程式

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階三次非線性方程式.

C. Complete the following table(10%)

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$2e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2 + 3.$

$dy = (2x^2+3)\,dx.$

$\int dy - \int (2x^2+3)\,dx = C.$

$y - \frac{2}{3}x^3 - 3x = C$ ✓

2. $(\cos x)dy = y\sin x\,dx$

$\frac{\cos x\,dy}{N} - \frac{y\sin x\,dx}{M} = 0$

~~$\Rightarrow \frac{dy}{y} = \frac{\sin x}{\cos x}dx$~~

~~$\frac{1}{y}dx - \frac{\sin x}{\cos x}dx = 0.$~~

~~$\int \frac{1}{y}dy - \int \frac{\sin x}{\cos x}dx = C$~~

~~$\ln y =$~~

$\frac{\partial M}{\partial y} = -\sin x$

$\frac{\partial N}{\partial x} = -\sin x$

$\int \cos x\,dy = y\cos x + C$

$-\int y\sin x\,dx = +y\cos x + C$

$y\cos x + C = 0$ ✓.

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$4x^3y^3\,dx = -3x^4y^2\,dy$

$\frac{4x^3}{3x^4}dx = -\frac{y^2}{y^3}\,dy$

$\frac{4}{3}x^{-1}dx = -y^{-1}\,dy$

$\frac{4}{3}x^{-1}dx + y^{-1}dy = 0.$

$\frac{4}{3}\int x^{-1}dx - \int y^{-1}dy = C$

$\frac{4}{3}\ln x - \ln y = C$ ✓

?

4. $\underset{M}{(x+3y)}dx + \underset{N}{(3x-5y+2)}dy = 0$

$\frac{\partial M}{\partial y} = 3. \quad \frac{\partial N}{\partial x} = 3.$

$\int (x+3y)\,dx = \frac{1}{2}x^2 + 3xy + C_1$

$\int (3x-5y+2)\,dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$\Rightarrow \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y + C = 0$ ✓.

5. $xyy' - y^2 + x^2 = 0$

$xy\frac{dy}{dx} - y^2 + x^2 = 0.$

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6. $\underset{M}{(x^3 + y^2 + 1)}dx + \underset{N}{xy}dy = 0$

$\frac{\partial M}{\partial y} = 2y \quad \frac{\partial N}{\partial x} = 1$ (非正合).

+1 $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{N} = \frac{1-2y}{xy}$

7. $y' + \frac{1}{x}y = x^2$

+1 $xy' + y = x^3$

$x\frac{dy}{dx} + y = x^3$

$x\,dy + y\,dx = x^3\,dx$

$x\,dy = (x^3 - y)\,dx$

33 出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二乙 學號：9940111F 姓名：曾益勳

A. Terminology (15%)

① independent variable, ② general solution, ③ initial condition

① 微分方程式：描述自變數與未知函數及自變數導數。不隨別的變數改變的變數。

② 通解：滿足 ODE 解中含有的導函數。

③ 初始條件：

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 三階一次 非線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階一次 非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階三次 線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	cosx	e^{2x}	e^{2x}
cosx	-sinx	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\int (2x^2+3)\,dx = \frac{2}{3}x^3 + 3x + C$

2. $(\cos x)dy = y \sin x\, dx$

$\frac{(\cos x)(\sin x)}{dx} = \frac{y}{dy}$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$\int 4x^3y^2\,dx = x^4y^2 + C$

$3x^4y^2\,dy = 12x^3y$

$= \frac{1}{2}x^3 + \frac{1}{4}y^4 + 2yC = 0$

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

$\int (x+3y)\,dx = \frac{1}{2}x^2 + \frac{3}{2}y^2 + C$

$(3x - 5y + 2)\,dy = 3 - 5y$

5. $xyy' - y^2 + x^2 = 0$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$(x^3+y^2+1)\,dy = \frac{1}{4}x^4 + \frac{1}{3}y^3 + 1 + C$

$xy\,dy = y$

7. $y' + \frac{1}{x}y = x^2$

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出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機二乙 學號：9940198 姓名：紀皇穎

A. Terminology (15%)

independent variable, general solution, initial condition

自變數，不隨別的變數而改變的變數

通解，為滿足微分方程式之關係式，亦即該微分方程式之原函數

初始條件，自變數指定在同一點的拘束條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 3階 1次 線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 2階 1次 非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 2階 3次 非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$\frac{1}{2}e^{2x}$ $e^{2x}\cdot 2$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$f(x)\,dx = g(y)\,dy$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$dy = (2x^2+3)\,dx$

$y = \frac{2}{3}x^3 + 3x + C$

2. $(\cos x)dy = y\sin x\,dx$

$\frac{dy}{y} = \frac{\sin x}{\cos x}\cdot dx$ 令 $u = \cos x$, $du = -\sin x\,dx$

左 $= \ln|y| + C$ 右 $= \int \frac{\sin x}{\cos x}dx = -\int \frac{du}{u} = -\ln|\cos x| + C$

$\ln|y| + \ln|\cos x| = \ln C$

$\ln(y\cdot\cos x) = \ln(C)$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$4x^3y^3dx = -3x^4y^2dy$ $x^4y^3 = C$ $u_2 = \int 3x^4y^2dy = x^4y^3 + C$

$4\frac{x^3}{x^4}dx = -3\frac{y^2}{y^3}dy$ $\frac{\partial M}{\partial y} = 4x^3(3y^2) = 12x^3y^2$

$4(\frac{1}{x}dx) = -3\frac{dy}{y}$ $\frac{\partial N}{\partial x} = 12x^3y^2$

$4\ln|x| = -3\ln|y| + C$ ⇒正合

$4\ln x + 3\ln y = C$ $u_1 = \int 4x^3y^3dx = x^4y^3 + C_1$

4. $(x+3y)dx + (3x-5y+2)dy = 0$

$\frac{\partial M}{\partial y} = 3$. $\frac{\partial N}{\partial x} = 3$ ⇒正合

$u_1 = \int (x+3y)dx = \frac{1}{2}x^2 + 3yx + C_1$

$u_2 = \int (3x-5y+2)dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$u = \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y + C$

5. $xyy' - y^2 + x^2 = 0$

$xy(\frac{dy}{dx}) + x^2 - y^2 = 0$

$xy\,dy + (x^2 - y^2)dx = 0$

$\frac{\partial M}{\partial y} = -2y$ $\frac{\partial N}{\partial x} = y$ ⇒非正合

令 $y = ux$

$y' = u'x + u$

$x(ux)(u'x+u) - u^2x^2 + x^2 = 0$

$xuu' + u^2 - u^2 + 1 = 0$

$xu(\frac{du}{dx}) = -1$

$u\,du = -\frac{dx}{x}$

$\frac{u^2}{2} = -\ln|x| + C$

$\frac{1}{2}(\frac{y^2}{x^2}) + \ln x = C$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$\frac{\partial M}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = y$

$x(x)(x^4 + xy^2 + x)dx + x^2y\,dy = 0$

$u_1 = \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C_1$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y-y}{N} = \frac{y}{xy} = \frac{1}{x}$ $u_2 = \frac{1}{2}x^2y^2 + C_2$

$I = e^{\int(\frac{1}{x})dx} = e^{\ln x} = x$

7. $y' + \frac{1}{x}y = x^2$

$I = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = x$

$y = \frac{1}{I}[\int IQ\,dx + C]$

$= \frac{1}{x}[\int (x)(x^2)dx + C]$

$= \frac{1}{x}[\int x^3dx + C]$

$= \frac{1}{x}[\frac{1}{4}x^4 + C] \Rightarrow y = \frac{1}{4}x^3 + \frac{C}{x}$

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出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二乙 學號：9964096 姓名：陳德勞

A. Terminology (15%)

independent variable, general solution, initial condition

independent variable 自變數
就是和其他的變數沒有太大的關係不影響人改變而改變的數

general solution 通解
滿足DE之代數函數關係式而且原函數跟DE之
-n階ODE的解包含n個任意常數

initial condition 邊界條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 3階0次 線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 2階1次 非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 2階3次 非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	cosx	e^{2x}	$2e^{2x}$
cosx	-sinx	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\int 1 = \int dx$

$\frac{dy}{dx} = 2x^2 + 3$

$\int f(x)dx = \int g(y)dy$

$M(x,y)dx + N(x,y)dy = 0$

$dy = (2x^2+3)dx$

$y = \frac{2}{3}x^3 + 3x + C$

2. $(\cos x)dy = y \sin x\, dx$

$\frac{dy}{y} = \frac{\sin x}{\cos x}dx$

左 $= \ln|y| + C$

右 $= \int \frac{\sin x}{\cos x}dx$

令 $u = \cos x + 1$

$du = -\sin x dx$

$= -\int \frac{du}{u} = -\ln|\cos x| + C$

$\ln|y| + \ln|\cos x| = \ln C$

$\ln(y \cdot \cos x) = \ln(C)$

$y\cos x = C$

$y = \frac{C}{\cos x}$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$5x^4\,4y^4\,dx + 4x^5\,2y^3 = 0$

$4x^3y^3dx = -3x^4y^2dy$

$4\frac{x^3}{x^4}dx = -3\frac{dy}{y}$

$4\ln|x| = -3\ln y = C$

$\ln x^4 + \ln y^3 = C$

$x^4y^3 = C$

4. $(x+3y)dx + (3x-5y+2)dy = 0$

$(1+3)dx + (3-5)dy = 0$

$\frac{\partial u}{\partial y} = 3 \Rightarrow \frac{\partial u}{\partial x} = 3$ ⇒正合

$u_1 = \int (x+3y)dx = \frac{1}{2}x^2 + 3yx + C$

$u_2 = \int (3x - 5y + 2)dx = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$u = \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y + C$

5. $xyy' - y^2 + x^2 = 0$

$xy(\frac{dy}{dx}) + x^2 - y^2 = 0$

$xy\,dy + (x^2 - y^2)dx = 0$

$\frac{\partial u}{\partial y} = -2y \quad \frac{\partial u}{\partial x} = y$ ⇒非正合

$y = ux \quad y' = u'x + u$

$x(ux)(u'x+u) - u^2x^2 + x^2 = 0$

$xuu' + u^2 - u^2 + 1 = 0$

$xu(\frac{du}{dx}) = -1$

$u\,du = -\frac{dx}{x}$

$\frac{u^2}{2} = \ln|x| + C$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$\frac{\partial u}{\partial y} = 2y \quad \frac{\partial v}{\partial x} = y$

$\frac{\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}}{v} = \frac{2y - y}{v} = \frac{y}{xy} = (\frac{1}{x})$

$I = e^{\int (\frac{1}{x})dx} = e^{\ln x} = x$

$x(x)(x^4 + xy^2 + y)dx + x^2 y\,dy = 0$

$u_1 = \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C_1$

$u_2 = \frac{1}{2}x^2y^2 + C$

7. $y' + \frac{1}{x}y = x^2$

$I = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx}$

$= x$

$y = \frac{1}{I}[\int IQ\,dx + C]$

$= \frac{1}{x}[\int (x)(x^2)dx + C]$

$= \frac{1}{x}[\int x^3\,dx + C]$

$= \frac{1}{x}[\frac{1}{4}x^4 + C]$ ⇒ $y = \frac{1}{4}x^3 + \frac{C}{x}$