100 出題負責教師：蔣祥第

建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機二甲 學號：9940124 姓名：杜宗安

A. Terminology (15%)

independent variable, general solution, initial condition

independent variable 自變數，不隨別的函數改變而改變之函數

general solution, GS 通解，滿足DE微分方程式的代數函數關係式

initial condition IC 初始條件

由通解GS求特解PS自變數(常為時間)被指定為某一點(常為0)之特定條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 三階一次線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階一次非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階三次非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$2e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2+3$

$\Rightarrow 2x^2+3 \cdot dx = dy$

$\int 2x^2+3 \cdot dx = \int 1 dy$

$= \frac{2}{3}x^3+3x+C_1 = y+C_2$ A: $\frac{2}{3}x^3+3x-y=C$ #

2. $(\cos x)dy = y \sin x\, dx$

$\frac{1}{y}dy = \frac{\sin x}{\cos x} dx$ $\cos x = u$, $du = -\sin x\, dx$

$\int \Rightarrow \int \frac{1}{y} dy = \int \frac{\sin x}{\cos x} \cdot dx$

$\ln(y)+c = -\ln(\cos x)+c$

$= \ln(y)+\ln(\cos x) = c$

$\Rightarrow y\cos x = C$ #

3. $4x^3y^3dx + 3x^4y^2dy = 0$ M N good!

$\frac{\partial M}{\partial y} = 12x^3y^2$ $\frac{\partial N}{\partial x} = 12x^3y^2$ 正合

$u_1 = \int 4x^3y^3 dx = x^4y^3 + C_1$

$u_2 = \int 3x^4y^2 dy = x^4y^3 + C_2$ ✓

$U = u_1 \cup u_2 \Rightarrow x^4y^3 + C = 0$ # or $u = x^4y^3 + C$

4. $(x+3y)dx + (3x-5y+2)dy = 0$ M N

$\frac{\partial M}{\partial y} = 3$ $\frac{\partial N}{\partial x} = 3$ 正合 ✓

$u_1 = \int x+3y\, dx = \frac{1}{2}x^2 + 3xy + C_1$

$u_2 = \int 3x-5y+2\, dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$u = u_1 \cup u_2 \Rightarrow \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y = C$ #

5. $xyy' - y^2 + x^2 = 0$

$y = ux$ $u = \frac{y}{x}$

$y' = u'x + u$

$x \cdot ux \cdot (u'x+u) - u^2x^2 + x^2 = 0$

$ux^2(u'x+u) - u^2x^2 + x^2 = 0$

$uu'x^3 + u^2x^2 - u^2x^2 + x^2 = 0$

$ux^3\frac{du}{dx} = -x^2$

$u \cdot du = -\frac{x^2}{x^3} \cdot dx$

$\int u\, du = \int -\frac{1}{x} dx$

$\Rightarrow \frac{u^2}{2} + C_1 = -\ln(x) + C_2$

$\Rightarrow \frac{y^2}{2x^2} + C_1 = -2\ln(x) + C_2$

$= \frac{y^2}{x^2} + \ln(x^2) = C$ #

6. $(x^3 + y^2x^2 + 1)dx + xydy = 0$ M N

$\frac{\partial M}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = y$ $u = \int x^4 + xy^2 + x \cdot dx$

$\frac{\partial M - \partial N}{N} = \frac{y}{xy} = \frac{1}{x} \cdot f(x)$

$u_1 = \int x^2y \cdot dy$

$\Rightarrow e^{\int \frac{1}{x} dx}$

$\Rightarrow e^{\ln(x)} = x$

$u_1 = \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C_1$

$u_2 = \frac{1}{2}x^2y^2 + C_2$

$u = u_1 \cup u_2 \Rightarrow \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C$

7. $y' + \frac{1}{x}y = x^2$

$\frac{dy}{dx} + \underbrace{\frac{1}{x}}_{P(x)} \cdot y = \underbrace{x^2}_{Q}$

$I = e^{\int P(x)\, dx} = e^{\ln(x)} = x$ ✓

$\Rightarrow \frac{1}{I}(\int IQx\, dx + C)$ $\Rightarrow \frac{1}{I}(\int IQx + C)$

$= \frac{1}{x}(\int x \cdot x^2 \cdot dx + C)$ $= \frac{1}{x}(\int x^3 \cdot dx + C)$

$= \frac{1}{x}(\frac{x^4}{4} + C)$ $= \frac{1}{x}(\frac{x^4}{4} + C)$

$= \frac{x^3}{4} + \frac{C}{x}$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二丙 學號：99401167 姓名：林冠廷

A. Terminology (15%)

independent variable, general solution, initial condition

1. Independent Variable 自變數 ✓
不隨別的函數改變而改變的函數。

2. General Solution 通解
為滿足微分方程式的代數函數關係式。

3. Initial Condition 初始條件 ✓
G.S求P.S自變數(時間)在指定某一點的拘束條件。

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

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(1) $y''' - 5xy' = e^x - 1$ 三階，一次，線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階，一次，非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階，三次，線性

C. Complete the following table(10%)

-1

f	f'	f	f'
sinx	$\cos x$	e^{2x}	$2\cdot e^{2x}$
cosx	$-\sin x$	x^3	$3x^{(2x)}$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2 + 3$

$\int dy = \int (2x^2+3)\,dx$

左：$\int dy = y + C_1$

右：$\int (2x^2+3)\,dx = \frac{2}{3}x^3 + 3x + C_2$

$y = \frac{2}{3}x^3 + 3x + C$ ✓ #

2. $(\cos x)dy = y\sin x\,dx$ $y = \frac{C}{\cos x}$

$\Rightarrow \int \frac{1}{y}dy = \int \frac{\sin x}{\cos x}dx \Longrightarrow$?

-6

左 $\int \frac{1}{y}dy = \ln|y| + C_1$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$\frac{\partial M}{\partial y} = 12x^3y^2$ $\frac{\partial N}{\partial x} = 12x^3y^2$ (正合)

$u_1 = \int 4x^3y^3\,dx = x^4y^3 + C_1$

$u_2 = \int 3x^4y^2\,dy = x^4y^3 + C_2$

$u_1 \cup u_2 = x^4y^3 + C = u$ # ✓

4. $(x+3y)dx + (3x-5y+2)dy = 0$

$\frac{\partial M}{\partial y} = 3$ $\frac{\partial N}{\partial x} = 3$ (正合)

$u_1 = \int (x+3y)\,dx = \frac{1}{2}x^2 + 3xy + C_1$

$u_2 = \int (3x-5y+2)\,dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$u_1 \cup u_2 = \frac{1}{2}x^2 + 3xy + 2y - \frac{5}{2}y^2 + C = u$ # ✓

5. $xyy' - y^2 + x^2 = 0$ $\frac{1}{2}(\frac{y^2}{x^2}) + \ln|x| = C$

令 $y = ux$ $y' = u'x + u$ $\int u\,du + \int \frac{1}{x}dx = 0$

$\Rightarrow ux(u'x+u) - u^2x^2 + x^2 = 0$ $\frac{1}{2}u^2 + \ln|x| = C$

$uxu' + u^2 - u^2 + 1 = 0$ $\frac{1}{2}\frac{y^2}{x^2} = (Cx^{-1})$?

$ux\frac{du}{dx} + 1 = 0$ $y^2 = 2Cx$ #

$udu + \frac{1}{x}dx = 0$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$u_1 = \int (x^4 + xy^2 + x)\,dx$

$= \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C_1$

$\frac{\partial M}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = y$ (非正合)

$u_2 = \int x^2y\,dy = \frac{1}{2}x^2y^2 + C_2$

$\frac{2y-y}{xy} = \frac{1}{x}$

$I = e^{\int \frac{1}{x}dx} = x$ $u = u_1 \cup u_2 = \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{2}x^2 + C$ #

$x(x^3 + y^2 + 1)dx + x(xy)dy = 0$

7. $y' + \frac{1}{x}y = x^2$

$P(x)$ $Q(x)$

$4y = x^3 + (4C)$ =#

$I = e^{\int \frac{1}{x}dx} = x$

$y = \frac{1}{x}(\int x\cdot x^2\,dx + C)$ $y = \frac{1}{4}x^3 + \frac{C}{x}$

$= \frac{1}{x}\cdot\frac{1}{4}x^4 + C$

$= \frac{1}{4}x^3 + C$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二丙 學號：9940103 姓名：郭建志

A. Terminology (15%)

independent variable, general solution, initial condition

independent variable：自變數，不隨別的函數而改變之函數

general solution：通解，滿足微分方程式的函數關係式

initial condition：初始條件，

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ ：三階一次，線性

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ ：二階一次，非線性

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ ：二階三次

C. Complete the following table(10%)

f	f'	f	f'
sinx	cos x	e^{2x}	$2e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$\frac{dy}{dx} = 2x^2 + 3 \Rightarrow dy = 2x^2 + 3\,dx$

$\int dy = \int 2x^2 + 3\,dx$

$y = \frac{2}{3}x^3 + 3x + C$

$\frac{2}{3}x^3 + 3x - y = -C$

2. $(\cos x)dy = y \sin x\,dx$

$\frac{dy}{y} = \frac{\sin x}{\cos x}\,dx$

$f_1 = \ln y$, f_2 = 令 $u = \cos x$, $du = -\sin x\,dx$

$-\int \frac{du}{u} = -\ln|u| = -\ln\cos x + C$

$\ln y = -\ln\cos x + C$

$\ln y + \ln\cos x = C$

$\ln(y\cos x) = C$

$\Rightarrow y\cos x = C$ $y = \frac{C}{\cos x}$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$4x^3y^3dx = -3x^4y^2dy$

$\frac{4x^3}{x^4}dx = \frac{-3y^2}{y^3}dy$

$\frac{4}{x}dx = \frac{-3}{y}dy$

$4\ln x = -3\ln y + C$

$4\ln x + 3\ln y = C$

$\ln x^4 + \ln y^3 = C$

$\ln(x^4y^3) = C$

$x^4y^3 = C$

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

M N

$\frac{\partial M}{\partial y} = 3$, $\frac{\partial N}{\partial x} = 3$ 正合

$u_1 = \int x + 3y\,dx = \frac{1}{2}x^2 + 3xy + C_1$

$u_2 = \int 3x - 5y + 2\,dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

$u = \frac{1}{2}x^2 + 3xy - \frac{5}{2}y^2 + 2y + C$

5. $xyy' - y^2 + x^2 = 0$

令 $y = ux$

$y' = u'x + u$

$x(ux)[u'x + u] - (ux)^2 + x^2 = 0$

$xuu'x + x^2 - x^2 + 1 = 0$

$xu\frac{du}{dx} = -1$

$udu = -\frac{dx}{x}$

$\frac{u^2}{2} = \frac{1}{2}(\frac{y^2}{x^2}) = -\ln x + C$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$\frac{\partial M}{\partial y} = 2y$, $\frac{\partial N}{\partial x} = y$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{y}{xy} = \frac{1}{x}$

$I = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$u_1 = \int x^4 + xy^2 + x^2\,dx$

$= \frac{1}{5}x^5 + \frac{1}{2}x^2y^2 + \frac{1}{3}x^3 + \cdots$

$u_2 = \int x^2y\,dy = \frac{1}{2}x^2y^2 + C$

$x^4 + xy^2 + x^2dx + x^2y\,dy = 0$ (正合) $u = \cdots$

7. $y' + \frac{-1}{x}y = x^2$

P Q

$\frac{dy}{dx} + \frac{1}{x}y = x^2 \Rightarrow \frac{dy}{dx} = x^2 - \frac{1}{x}y$

$I = e^{\int P(x)dx}$

$dy = (x^2 - \frac{1}{x}y)dx$

$\int dy = \int x^2 - \frac{1}{x}y\,dx$

$y = \frac{1}{3}x^3 - \ln x\,y + C$

$\frac{1}{3}x^2 - \ln x\,y - y = -C$

$y = \frac{1}{I}[\int IQ\,dx + C]$

$= \frac{1}{I}[$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二丙 學號：9940116 姓名：沈昱宏

A. Terminology (15%)

independent variable, general solution, initial condition

(1) 自變數
不隨別的函數改變而改變的函數

(3) 通解：
為滿足微分方程式之代數函數之關係式

(3) 初始條件
Gs求PS自變數時間在指定之約束條件

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$ 三階一次方程式(線性)

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$ 二階一次方程式

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$ 二階三次方程式非線性

C. Complete the following table(10%)

f	f'	f	f'
sinx	cosx	e^{2x}	$2e^{2x}$ / $2e^{2x}$
cosx	$-\sin x$	x^3	$3x^2$

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$ (1) $f(x)dx = g(y)dy$

$\frac{dy}{dx} = 2x^2 + 3$

$\Rightarrow dy = (2x^2 + 3)dx$

$\int dy = \int (2x^2 + 3)dx + C$

$y = \frac{2}{3}x^3 + 3x + C$ ✓

2. $(\cos x)dy = y \sin x\, dx$

$\int (\cos x)dy = \int y(\sin x)dx$

$0 = -y\cos x + C$

$= -y\cos x + C$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

=)

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

5. $xyy' - y^2 + x^2 = 0$

$xy\frac{dy}{dx} - y^2 + x^2 = 0$

$xy\frac{dy}{dx} = y^2 - x^2$

$\Rightarrow xy\,dy = (y^2 - x^2)dx$

=)

6. $(x^3 + y^2 + 1)dx + xydy = 0$

7. $y' + \frac{1}{x}y = x^2$

$\frac{dy}{dx} + \ln y = x^2$

$\Rightarrow \frac{dy}{dx} = x^2 - \ln y$

$\Rightarrow dy = (x^2 - \ln y)dx$

$\int dy = \int (x^2 - \ln y)dx$

$\frac{1}{3}x^3 + C$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：机二 丙 學號：99401183 姓名：馮智遠

A. Terminology (15%)

independent variable, general solution, initial condition

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$

C. Complete the following table(10%)

f	f'	f	f'
sinx	cosx	e^{2x}	$2e^{2x}$
cosx	-sinx	x^3	$3x^2$

+9

D, Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

2. $(cosx)dy = y\ sinx\ dx$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

5. $xyy' - y^2 + x^2 = 0$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

7. $y' + \frac{1}{x}y = x^2$

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建國科技大學 一百學年度第一學期 班級：機二 科目： 工程數學（一） (期末考) 考試試題

** 註：關書、不可使用計算機 班級：機械二丙 學號：9940117l0 姓名：廖思政

A. Terminology (15%)

independent variable, general solution, initial condition

自變數：不會因別的函數改變而改變

十五

階數：

B. Please determine the order, degree, and linear or non-linear of the following equations. (15%)

(1) $y''' - 5xy' = e^x - 1$

(2) $y(\frac{d^2x}{dy^2}) = y^2 + 1$

(3) $(\frac{d^2y}{dx^2})^3 + 2(\frac{dy}{dx}) = 3y$

C. Complete the following table(10%)

f	f'	f	f'
sinx		e^{2x}	
cosx		x^3	

D. Solve the following differential equations.(70%)

1. $y' = 2x^2 + 3$

$y' = u'x + 1$ $\begin{cases} f(x)dx = g(y)dy \\ M(x,y)dx + N(x,y)dy = 0 \end{cases}$

$dy = (2x^2+3)dx$

$\int \Rightarrow y = \frac{2}{3}x^3 + 3y + C$

2. $(\cos x)dy = y \sin x\, dx$

$\frac{dy}{y} = \frac{\sin x}{\cos x}dx$

左 $= \ln y$

右 $= \int \frac{\sin x}{\cos x}dx$

$u = \cos x$

$du = -\sin x\, dx$

$= -\int \frac{du}{u} = -\ln|u| = -\ln\cos x + C$

$\ln y = -\ln\cos x + C$

$\ln y + \ln\cos x = C$

$\ln(y\cos x) = C$

$\Rightarrow y\cos x = C$

$y = \frac{C}{\cos x}$

3. $4x^3y^3dx + 3x^4y^2dy = 0$

$4x^3y^3dx = -3x^4y^2dy$

$\frac{4x^3}{x^4}dx = \frac{-3y^2}{y^3}dy$

$\frac{4}{x}dx = \frac{-3}{y}dy$

$4\ln x = -3\ln y + C$

$4\ln x + 3\ln y = C$

$\ln x^4 + \ln y^3 = C$

$\ln(x^4y^3) = C$

$\Rightarrow x^4y^3 = C$

4. $(x + 3y)dx + (3x - 5y + 2)dy = 0$

$u_1 = \int (x+3y)dx = \frac{1}{2}x^2 + 3yx + C_1$

$u_2 = \int (3x - 5y + 2)dy = 3xy - \frac{5}{2}y^2 + 2y + C_2$

5. $xyy' - y^2 + x^2 = 0$

$x^2(\frac{dy}{dx}) - y^2 + x^2 = 0$

$xy\,dy + (x^2 - y^2)dx = 0$

$\frac{dM}{dy} = -2y,\ \frac{dN}{dx} = y$

$y = ux,\ y' = u'x + u$

$x(ux)(u'x + u) - (u^2x^2) + x^2 = 0$

$xuu' + u^2 - u^2 + 1 = 0$

$xu = \frac{du}{dx} = -1$

$u\,du = -\frac{dx}{x}$

$\frac{u^2}{2} = \frac{1}{2}(\frac{y^2}{x^2}) = -\ln x + C$

6. $(x^3 + y^2 + 1)dx + xydy = 0$

$I = e^{\int (\frac{1}{x})dx} = e^{\ln x} = x$

$(x^4e^{xy^2} + x)dx + x^2y\,dy = 0$ 正合

$u_1 = \int (x^4 + xy^2 + x)'dx$

7. $y' + \frac{1}{x}y = x^2$

$I = e^{\int P(x)dx}$

$y = \frac{1}{I}[\int Iq\,dx + C]$

$= \frac{1}{x}[\frac{1}{4}x^4 + C]$

$= \frac{1}{I}[\int (x)(x^2)dx + C]$

$y = \frac{1}{4}x^3 + \frac{C}{x}$

$= \frac{1}{I}[\int x^3dx + C]$