出題教師 :劉政湖

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建國科技大學 97 學年度第 一 學期 四技機二乙 科目: 工程數學 （期末考）考試試題 班級: 機械二乙 學號: 9640108 姓名: 詹邦王

(每題 15%)

1. Find the transpose and the inverse of the matrix A

$$A=\begin{bmatrix}1&0&0\\2&3&0\\0&5&-1\end{bmatrix}$$

$|A|=(-3)-(0)=-3$

$[(-1)^{ij}M_{ij}]=\begin{bmatrix}-3&2&10\\0&-1&-5\\0&0&3\end{bmatrix}$

$A^{-1}=\frac{1}{-3}\begin{bmatrix}-3&0&0\\2&-1&0\\10&-5&3\end{bmatrix}$ ※

2Find an equation of a plane that contains (0,0,0),(1,1,1),(1,3,-1)

① $\begin{vmatrix}X&Y&Z\\1&3&-1\\1&1&1\end{vmatrix}=3X-Y+Z$

$-(3Z+Y-X)$

$=4X-2Y-2Z$

$=2X-Y-Z$ ※

3.Find symmetric equations of the line through (1,4,9),(10,14,-2)

③ $\begin{vmatrix}X&Y&Z\\1&4&9\\10&14&-2\end{vmatrix}=-8X$

4 Evaluate the rank of the matrix

$$A=\begin{bmatrix}1&2&-1\\2&4&6\\0&0&-8\end{bmatrix}=\begin{bmatrix}1&2&-1\\0&0&8\\0&0&-8\end{bmatrix}$$ Rank(2) ※

5.Use Gaussian elimination to solve

$$\begin{cases}x_1-x_2-x_3=8\\x_1-x_2+x_3=3\\-x_1+x_2+x_3=4\end{cases}$$

$$\begin{bmatrix}1&-1&-1&8\\1&-1&1&3\\-1&1&1&4\end{bmatrix}=\begin{bmatrix}1&-1&-1&8\\0&0&2&-5\\0&0&0&12\end{bmatrix}$$

※

6. Determine the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}2&2\\3&1\end{bmatrix}$

⑥ $|A-\lambda I|=0$

$\begin{bmatrix}2-\lambda&2\\3&1-\lambda\end{bmatrix}=0$

$(2-\lambda)(1-\lambda)-6=0$

$\lambda^2-3\lambda+2-6=0$

$\lambda^2-3\lambda-4=0$

$\lambda=4,-1$

① $\lambda=4$

$\begin{bmatrix}-2&2\\3&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}1\\1\end{bmatrix}$

② $\lambda=-1$

$\begin{bmatrix}3&2\\3&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}2\\-3\end{bmatrix}$

#

7 $A=\begin{bmatrix}2&2\\3&1\end{bmatrix}$ Evaluate $A^{100}=?$

⑦ $|A-\lambda I|=0$

$\begin{bmatrix}2-\lambda&2\\3&1-\lambda\end{bmatrix}=0$

$\lambda=4,-1$

① $\lambda=4$

$\begin{bmatrix}-2&2\\3&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}1\\1\end{bmatrix}$

② $\lambda=-1$

$\begin{bmatrix}3&2\\3&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}\to\begin{bmatrix}2\\-3\end{bmatrix}$

PAP^{-1} ①

$P=\begin{bmatrix}2&1\\-3&1\end{bmatrix}$

$PAP^{-1}=\frac{1}{5}\begin{bmatrix}1&-1\\3&2\end{bmatrix}\begin{bmatrix}2&2\\3&1\end{bmatrix}\begin{bmatrix}2&1\\-3&1\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}-1&1\\12&8\end{bmatrix}\begin{bmatrix}2&1\\-3&1\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}-5&0\\0&20\end{bmatrix}=\begin{bmatrix}-1&0\\0&4\end{bmatrix}$

$PP^{100}P^{-1}$

$=\frac{1}{5}\begin{bmatrix}2&1\\-3&1\end{bmatrix}\begin{bmatrix}1&0\\0&4^{100}\end{bmatrix}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2&4^{100}\\-3&4^{100}\end{bmatrix}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2+3\cdot4^{100}&-2+2\cdot4^{100}\\-3+3\cdot4^{100}&3+2\cdot4^{100}\end{bmatrix}$

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(每題 15%)

1. Find the transpose and the inverse of the matrix A

$$A=\begin{bmatrix}1 & 0 & 0\\ 2 & 3 & 0\\ 0 & 5 & -1\end{bmatrix}$$

2Find an equation of a plane that contains (0,0,0),(1,1,1),(1,3,-1)

3.Find symmetric equations of the line through (1,4,9),(10,14,-2)

4 Evaluate the rank of the matrix

$$A=\begin{bmatrix}1 & 2 & -1\\ 2 & 4 & 6\\ 0 & 0 & -8\end{bmatrix}$$

5.Use Gaussian elimination to solve $\begin{cases} x_1 - x_2 - x_3 = 8 \\ x_1 - x_2 + x_3 = 3 \\ -x_1 + x_2 + x_3 = 4 \end{cases}$

6. Determine the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix}2 & 2\\ 3 & 1\end{bmatrix}$

7 $A=\begin{bmatrix}2 & 2\\ 3 & 1\end{bmatrix}$ Evaluate $A^{100}=?$

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60

(每題 15%)

1. Find the transpose and the inverse of the matrix A

$$A=\begin{bmatrix}1&0&0\\2&3&0\\0&5&-1\end{bmatrix}$$

2. $\begin{vmatrix}x&y&z\\1&1&1\\1&3&-1\end{vmatrix}=0$

$-x+3z+y-z-3x+y=0$

$-4x+2y+2z=0$ #

2Find an equation of a plane that contains (0,0,0),(1,1,1),(1,3,-1)

3.Find symmetric equations of the line through (1,4,9),(10,14,-2)

3. $m=\frac{-10}{-9}=\frac{10}{9}$

$(y-4)=\frac{10}{9}(x-1)$

$y-4=\frac{10}{9}x-\frac{10}{9}$

$9y-36=10x-10$

$10x-9y+26=0$ #

4 Evaluate the rank of the matrix

$$A=\begin{bmatrix}1&2&-1\\2&4&6\\0&0&-8\end{bmatrix}\xrightarrow{\times 2}\begin{bmatrix}1&2&-1\\0&0&-8\\0&0&-8\end{bmatrix}$$

Rank(A) = 2

RA RA Rank

5.Use Gaussian elimination to solve

$$\begin{cases}x_1-x_2-x_3=8\\x_1-x_2+x_3=3\\-x_1+x_2+x_3=4\end{cases}$$

$\begin{bmatrix}1&-1&-1&8\\1&-1&1&3\\-1&1&1&4\end{bmatrix}\xrightarrow{\times -1}$

$\begin{bmatrix}1&-1&-1&8\\0&0&2&5\\-1&1&1&4\end{bmatrix}\xrightarrow{\times 1}$

$\begin{bmatrix}1&-1&-1&8\\0&0&2&5\\0&0&0&12\end{bmatrix}$

$=-2\begin{vmatrix}0&0&12\\-1&-1&8\\0&2&5\end{vmatrix}$

$\begin{bmatrix}1&-1&-1&8\\0&0&2&5\\-1&1&1&4\end{bmatrix}\xrightarrow{\times 1}$

$\begin{bmatrix}0&0&0&12\\1&-1&-1&8\\0&0&2&5\end{bmatrix}$

$=-2\cdot(-12)=24$

6. Determine the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}2&2\\3&1\end{bmatrix}$

$|A-\lambda I|=\begin{bmatrix}2-\lambda&2\\3&1-\lambda\end{bmatrix}$

$(2-\lambda)(1-\lambda)-6=0$

$2-2\lambda-\lambda+\lambda^2-6=0$

$\lambda^2-3\lambda-4=0$

$(\lambda+1)(\lambda-4)=0$

$\lambda=-1,4$

$\lambda=-1$

$\begin{bmatrix}3&2\\3&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}2\\-3\end{bmatrix}$

$\lambda=4$

$\begin{bmatrix}-2&2\\3&-3\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

7 A $=\begin{bmatrix}2&2\\3&1\end{bmatrix}$ Evaluate $A^{100}=?$

$PD^{100}P^{-1}=\frac{1}{5}\begin{bmatrix}2&1\\-3&1\end{bmatrix}\begin{bmatrix}1&0\\0&4^{100}\end{bmatrix}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2&4^{100}\\-3&4^{100}\end{bmatrix}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2+3\cdot4^{100}&-2+3\cdot4^{100}\\-3+3\cdot4^{100}&3+2\cdot4^{100}\end{bmatrix}$

$P=\begin{bmatrix}2&1\\-3&1\end{bmatrix}=5$

$P^{-1}=\frac{1}{5}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$P^{-1}AP=\frac{1}{5}\begin{bmatrix}1&-1\\3&2\end{bmatrix}\begin{bmatrix}2&2\\3&1\end{bmatrix}\begin{bmatrix}2&1\\-3&1\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}-1&1\\12&8\end{bmatrix}\begin{bmatrix}2&1\\-3&1\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}-5&0\\0&20\end{bmatrix}$

$=\begin{bmatrix}-1&0\\0&4\end{bmatrix}=D$

$3x_1+2x_2=0$

2 −3

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60

(每題 15%)

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(每題 15%)

1. Find the transpose and the inverse of the matrix A

$$A=\begin{bmatrix}1&0&0\\2&3&0\\0&5&-1\end{bmatrix}$$

1、 $A^{-1}=\frac{1}{|A|}[(-1)^{i+j}M_{ij}]$ ∴ $A^{-1}=-\frac{1}{3}\begin{bmatrix}-3&0&0\\2&-1&0\\10&-5&3\end{bmatrix}$ #

$|A|\begin{vmatrix}1&0&0\\2&3&0\\0&5&-1\end{vmatrix}=-3$

$[M_{ij}]=\begin{bmatrix}-3&-2&10\\0&-1&5\\0&0&3\end{bmatrix}$

$[(-1)^{i+j}M_{ij}]=\begin{bmatrix}-3&2&10\\0&-1&-5\\0&0&3\end{bmatrix}$

5

2Find an equation of a plane that contains (0,0,0),(1,1,1),(1,3,-1)

3.Find symmetric equations of the line through (1,4,9),(10,14,-2)

4 Evaluate the rank of the matrix

$$A=\begin{bmatrix}1&2&-1\\2&4&6\\0&0&-8\end{bmatrix}$$

2、 $\vec{a}=(1,3,-1)$, $\vec{b}=(4,4,2)$, $\vec{c}=(1,1,1)$

4、∴ Rank(A) = 0

5.Use Gaussian elimination to solve $\begin{cases}x_1-x_2-x_3=8\\x_1-x_2+x_3=3\\-x_1+x_2+x_3=4\end{cases}$

5、無解 ?

6. Determine the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}2&2\\3&1\end{bmatrix}$

6. $|A-\lambda_2|=\begin{vmatrix}2-\lambda&2\\3&1-\lambda\end{vmatrix}=0$ $D\chi=-1$

$\begin{bmatrix}x_1\\x_2\end{bmatrix}\rightarrow\begin{bmatrix}k_2\\-3\end{bmatrix}$

7 $A=\begin{bmatrix}2&2\\3&1\end{bmatrix}$ Evaluate $A^{100}=?$

7. $A^{100}=PD^{100}P^{-1}$

$=\frac{1}{5}\begin{bmatrix}2&1\\-3&1\end{bmatrix}\begin{bmatrix}1&0\\0&4^{100}\end{bmatrix}$

$\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2&4^{100}\\-3&4^{100}\end{bmatrix}\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix}2+3\cdot4^{100}&-2+2\cdot4^{100}\\-3+3\cdot4^{100}&3+2\cdot4^{100}\end{bmatrix}$

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